

SOME INEQUALITIES FOR DIFFERENTIABLE CONVEX FUNCTIONS WITH APPLICATIONS

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ABSTRACT. In this paper, the Authors establish a new identity for differentiable functions. By the well-known Hölder and power mean inequality, they obtain some integral inequalities related to the convex functions and apply these inequalities to special means.

1. INTRODUCTIONS

A function $f : I \rightarrow \mathbb{R}$ is said to be convex on I if inequality

$$(1.1) \quad f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

holds for all $x, y \in I$ and $t \in [0, 1]$. We say that f is concave if $(-f)$ is convex.

Geometrically, this means that if P, Q and R are three distinct points on the graph of f with Q between P and R , then Q is on or below the chord PR .

Theorem 1. *The Hermite-Hadamard inequality:* Let $f : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a convex function and $a, b \in I$ with $a < b$. The following double inequality:

$$(1.2) \quad f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x) dx \leq \frac{f(a) + f(b)}{2}$$

is known in the literature as Hadamard's inequality (or Hermite-Hadamard inequality) for convex functions. If f is a positive concave function, then the inequality is reversed.

In [5], Dragomir and Agarwal obtained inequalities for differentiable convex mappings which are connected to Hadamard's inequality, as follow:

Theorem 2. Let $f : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° , where $a, b \in I$, with $a < b$. If $|f'|^q$ is convex on $[a, b]$, then the following inequality holds:

$$(1.3) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{8} [|f'(a)| + |f'(b)|].$$

In [15], Pearce and Pečarić obtained inequalities for differentiable convex mappings which are connected with Hadamard's inequality, as follow:

Theorem 3. Let $f : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be differentiable mapping on I° , where $a, b \in I$, with $a < b$. If $|f'|^q$ is convex on $[a, b]$ for some $q \geq 1$, then the following inequality

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holds:

$$(1.4) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{4} \left(\frac{(|f'(a)|^q + |f'(b)|^q)}{2} \right)^{\frac{1}{q}}.$$

If $|f'|^q$ is concave on $[a, b]$ for some $q \geq 1$, then

$$(1.5) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{4} \left| f' \left(\frac{a+b}{2} \right) \right|.$$

In [1], Alomari, Darus and Kirmacı obtained inequalities for differentiable s -convex and concave mappings which are connected with Hadamard's inequality, as follow:

Theorem 4. Let $f : I \subseteq [0, \infty) \rightarrow \mathbb{R}$ be differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$. If $|f'|^q$, $q \geq 1$ is concave on $[a, b]$ for some fixed $s \in (0, 1]$, then the following inequality holds:

$$(1.6) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \left(\frac{b-a}{4} \right) \left(\frac{q-1}{2q-1} \right)^{1-\frac{1}{q}} \left[\left| f' \left(\frac{3a+b}{4} \right) \right| + \left| f' \left(\frac{a+3b}{4} \right) \right| \right].$$

For recent results and generalizations concerning Hadamard's inequality and concepts of convexity and concavity see [1]-[21] and the references therein.

Throughout this paper we will use the following notations and conventions. Let $J = [0, \infty) \subset \mathbb{R} = (-\infty, +\infty)$, and $a, b \in J$ with $0 < a < b$ and $f' \in L[a, b]$ and

$$\begin{aligned} A(a, b) &= \frac{a+b}{2}, \quad G(a, b) = \sqrt{ab}, \quad I(a, b) = \frac{1}{e} \left(\frac{b^b}{a^a} \right)^{\frac{1}{b-a}}, \\ H(a, b) &= \frac{2ab}{a+b}, \quad L(a, b) = \frac{b-a}{\ln b - \ln a} \\ L_p(a, b) &= \left(\frac{b^{p+1} - a^{p+1}}{(p+1)(b-a)} \right)^{1/p}, \quad a \neq b, \quad p \in \mathbb{R}, \quad p \neq -1, 0 \end{aligned}$$

be the arithmetic, geometric, identric, harmonic, logarithmic, generalized logarithmic mean for $a, b > 0$ respectively.

The main purpose of this paper is to establish refinements of Hadamard's inequality for convex functions.

2. MAIN RESULTS

In order to establish our main results, we first establish the following lemma.

Lemma 1. Let $f : J \rightarrow \mathbb{R}$ be a differentiable function on J° . If $f' \in L[a, b]$, then

$$\begin{aligned} & \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f \left(\frac{a+b}{2} \right) \\ &= \frac{1}{4} \int_0^1 (tb + (1-t)a) f' \left(\frac{1-t}{2}b + \frac{1+t}{2}a \right) dt \\ & \quad + \frac{1}{4} \int_0^1 (ta + (1-t)b) f' \left(\frac{1-t}{2}a + \frac{1+t}{2}b \right) dt \end{aligned}$$

for each $t \in [0, 1]$ and $a, b \in J$.

Proof. Integrating by parts, we get

$$\begin{aligned}
& \frac{1}{4} \int_0^1 (tb + (1-t)a) f' \left(\frac{1-t}{2}b + \frac{1+t}{2}a \right) dt \\
& + \frac{1}{4} \int_0^1 (ta + (1-t)b) f' \left(\frac{1-t}{2}a + \frac{1+t}{2}b \right) dt \\
= & (tb + (1-t)a) \left. \frac{f(\frac{1-t}{2}b + \frac{1+t}{2}a)}{\frac{a-b}{2}} \right|_0^1 - \int_0^1 \frac{f(\frac{1-t}{2}b + \frac{1+t}{2}a)}{\frac{a-b}{2}} (b-a) dt \\
& + (ta + (1-t)b) \left. \frac{f(\frac{1-t}{2}a + \frac{1+t}{2}b)}{\frac{b-a}{2}} \right|_0^1 - \int_0^1 \frac{f(\frac{1-t}{2}a + \frac{1+t}{2}b)}{\frac{b-a}{2}} (a-b) dt \\
= & \frac{bf(a) - af(\frac{a+b}{2})}{\frac{a-b}{2}} + 2 \int_0^1 f(\frac{1-t}{2}b + \frac{1+t}{2}a) dt \\
& + \frac{af(b) - bf(\frac{a+b}{2})}{\frac{b-a}{2}} + 2 \int_0^1 f(\frac{1-t}{2}a + \frac{1+t}{2}b) dt \\
= & \frac{bf(a) - af(\frac{a+b}{2})}{\frac{a-b}{2}} + \frac{4}{b-a} \int_a^{\frac{a+b}{2}} f(x) dx + \frac{af(b) - bf(\frac{a+b}{2})}{\frac{b-a}{2}} + \frac{4}{b-a} \int_{\frac{a+b}{2}}^b f(x) dx \\
= & \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right)
\end{aligned}$$

□

Theorem 5. Let $f : J \rightarrow \mathbb{R}$ be a differentiable function on J° . If $|f'|$ is convex on J , then

$$\begin{aligned}
(2.1) \quad & \left| \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right) \right| \\
& \leq \left(\frac{5}{48}a + \frac{7}{48}b \right) |f'(a)| + \left(\frac{7}{48}a + \frac{5}{48}b \right) |f'(b)|
\end{aligned}$$

for each $a, b \in J$.

Proof. Using Lemma 1 and from properties of modulus, and since $|f'|$ is convex on J , then we obtain

$$\begin{aligned}
& \left| \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right) \right| \\
& \leq \frac{1}{4} \int_0^1 (tb + (1-t)a) \left| f'\left(\frac{1-t}{2}b + \frac{1+t}{2}a\right) \right| dt \\
& \quad + \frac{1}{4} \int_0^1 (ta + (1-t)b) \left| f'\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right) \right| dt \\
& \leq \frac{1}{4} \int_0^1 (tb + (1-t)a) \left[\frac{1-t}{2} |f'(b)| + \frac{1+t}{2} |f'(a)| \right] dt \\
& \quad + \frac{1}{4} \int_0^1 (ta + (1-t)b) \left[\frac{1-t}{2} |f'(a)| + \frac{1+t}{2} |f'(b)| \right] dt \\
& = \frac{1}{4} \left(\frac{1}{6}a + \frac{1}{12}b \right) (|f'(b)|) + \frac{1}{4} \left(\frac{1}{3}a + \frac{5}{12}b \right) (|f'(a)|) \\
& \quad + \frac{1}{4} \left(\frac{1}{12}a + \frac{1}{6}b \right) (|f'(a)|) + \frac{1}{4} \left(\frac{5}{12}a + \frac{1}{3}b \right) (|f'(b)|) \\
& = \left(\frac{5}{48}a + \frac{7}{48}b \right) |f'(a)| + \left(\frac{7}{48}a + \frac{5}{48}b \right) |f'(b)|.
\end{aligned}$$

The proof is completed. \square

Proposition 1. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\begin{aligned}
& \left| \frac{1}{L(a, b)} - \frac{1}{H(a, b)} - \frac{1}{2A(a, b)} \right| \\
& \leq \left(\frac{5}{48}a + \frac{7}{48}b \right) \frac{1}{a^2} + \left(\frac{7}{48}a + \frac{5}{48}b \right) \frac{1}{b^2}
\end{aligned}$$

Proof. The proof follows from (2.1) applied to the convex function $f(x) = 1/x$. \square

Proposition 2. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\begin{aligned}
& \left| L_n^n(a, b) + \frac{(n-1)G^2(a, b)L_{n-1}^{n-1}(a, b)}{2} - \frac{1}{2}A^n(a, b) \right| \\
& \leq \frac{5n}{24}A(a^n, b^n) + \frac{7n}{24}A(ba^{n-1}, ab^{n-1})
\end{aligned}$$

Proof. The proof follows from (2.1) applied to the convex function $f(x) = x^n$, $n \geq 2$. \square

Proposition 3. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\left| -\ln I(a, b) + \frac{\ln(a^b/b^a)}{2(b-a)} + \frac{1}{2} \ln A(a, b) \right| \leq \frac{12}{48} + \frac{7b}{48a} + \frac{5a}{48b}$$

Proof. The proof follows from (2.1) applied to the convex function $f(x) = -\ln x$. \square

Theorem 6. Let $f : J \rightarrow \mathbb{R}$ be a differentiable function on J° . If $|f'|^q$ is convex on $[a, b]$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$, then

$$(2.2) \quad \left| \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right) \right| \\ \leq \frac{1}{4^{1+1/q}} L_p(a, b) \left[[|f'(b)|^q + 3|f'(a)|^q]^{\frac{1}{q}} + [|f'(a)|^q + 3|f'(b)|^q]^{\frac{1}{q}} \right]$$

Proof. From Lemma 1 and using the well-known Hölder integral inequality, we obtain

$$\begin{aligned} & \left| \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right) \right| \\ & \leq \frac{1}{4} \int_0^1 (tb + (1-t)a) \left| f'\left(\frac{1-t}{2}b + \frac{1+t}{2}a\right) \right| dt \\ & \quad + \frac{1}{4} \int_0^1 (ta + (1-t)b) \left| f'\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right) \right| dt \\ & \leq \frac{1}{4} \int_0^1 ((tb + (1-t)a)^p dt)^{\frac{1}{p}} \left[\int_0^1 \left| f'\left(\frac{1-t}{2}b + \frac{1+t}{2}a\right) \right|^q dt \right]^{\frac{1}{q}} \\ & \quad + \frac{1}{4} \int_0^1 ((ta + (1-t)b)^p dt)^{\frac{1}{p}} \left[\int_0^1 \left| f'\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right) \right|^q dt \right]^{\frac{1}{q}} \\ & \leq \frac{1}{4} \left(\frac{b^{p+1} - a^{p+1}}{(b-a)(p+1)} \right)^{\frac{1}{p}} \left[|f'(b)|^q \int_0^1 \frac{1-t}{2} dt + |f'(a)|^q \int_0^1 \frac{1+t}{2} dt \right]^{\frac{1}{q}} \\ & \quad + \frac{1}{4} \left(\frac{b^{p+1} - a^{p+1}}{(b-a)(p+1)} \right)^{\frac{1}{p}} \left[|f'(a)|^q \int_0^1 \frac{1-t}{2} dt + |f'(b)|^q \int_0^1 \frac{1+t}{2} dt \right]^{\frac{1}{q}} \\ & = \frac{1}{4^{1+1/q}} L_p(a, b) \left[[|f'(b)|^q + 3|f'(a)|^q]^{\frac{1}{q}} + [|f'(a)|^q + 3|f'(b)|^q]^{\frac{1}{q}} \right]. \end{aligned}$$

The proof is completed. \square

Proposition 4. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\left| \frac{1}{L(a, b)} - \frac{1}{H(a, b)} - \frac{1}{2A(a, b)} \right| \\ \leq \frac{1}{4^{1+1/q}} L_p(a, b) \left[[b^{-2q} + 3a^{-2q}]^{\frac{1}{q}} + [a^{-2q} + 3b^{-2q}]^{\frac{1}{q}} \right]$$

Proof. The proof follows from (2.2) applied to the convex function $f(x) = 1/x$. \square

Proposition 5. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\left| L_n^n(a, b) + \frac{(n-1)G^2(a, b)L_{n-1}^{n-1}(a, b)}{2} - \frac{1}{2} A^n(a, b) \right| \\ \leq \frac{1}{4^{1+1/q}} L_p(a, b) \left[[nb^{(n-1)q} + 3na^{(n-1)q}]^{\frac{1}{q}} + [na^{(n-1)q} + 3nb^{(n-1)q}]^{\frac{1}{q}} \right]$$

Proof. The proof follows from (2.2) applied to the convex function $f(x) = x^n$, $n \geq 2$. \square

Proposition 6. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\begin{aligned} & \left| -\ln I(a, b) + \frac{\ln(a^b/b^a)}{2(b-a)} + \frac{1}{2} \ln A(a, b) \right| \\ & \leq \frac{1}{4^{1+1/q}} L_p(a, b) \left[[b^{-q} + 3a^{-q}]^{\frac{1}{q}} + [a^{-q} + 3b^{-q}]^{\frac{1}{q}} \right] \end{aligned}$$

Proof. The proof follows from (2.2) applied to the convex function $f(x) = -\ln x$. \square

Theorem 7. Let $f : J \rightarrow \mathbb{R}$ be a differentiable function on J° . If $|f'|^q$ is convex on $[a, b]$ and $q \geq 1$, then

$$\begin{aligned} (2.3) \quad & \left| \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right) \right| \\ & \leq \frac{A^{1-\frac{1}{q}}(a, b)}{4 \times 12^{\frac{1}{q}}} \left\{ [|f'(b)|^q (2a+b) + |f'(a)|^q (4a+5b)]^{\frac{1}{q}} \right. \\ & \quad \left. + [|f'(a)|^q (a+2b) + |f'(b)|^q (5a+4b)]^{\frac{1}{q}} \right\} \end{aligned}$$

Proof. From Lemma 1 and using the well-known power mean inequality and since $|f'|^q$ is convex on $[a, b]$, we have

$$\begin{aligned} & \left| \frac{1}{b-a} \int_a^b f(x) dx + \frac{af(b) - bf(a)}{2(b-a)} - \frac{1}{2} f\left(\frac{a+b}{2}\right) \right| \\ & \leq \frac{1}{4} \left(\int_0^1 (tb + (1-t)a) dt \right)^{1-\frac{1}{q}} \left[\int_0^1 (tb + (1-t)a) \left| f'\left(\frac{1-t}{2}b + \frac{1+t}{2}a\right) \right|^q dt \right]^{\frac{1}{q}} \\ & \quad + \frac{1}{4} \left(\int_0^1 (ta + (1-t)b) dt \right)^{1-\frac{1}{q}} \left[\int_0^1 (ta + (1-t)b) \left| f'\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right) \right|^q dt \right]^{\frac{1}{q}} \\ & \leq \frac{1}{4} \left(\int_0^1 (tb + (1-t)a) dt \right)^{1-\frac{1}{q}} \left[\int_0^1 (tb + (1-t)a) \left(\frac{1-t}{2} |f'(b)|^q + \frac{1+t}{2} |f'(a)|^q \right) dt \right]^{\frac{1}{q}} \\ & \quad + \frac{1}{4} \left(\int_0^1 (ta + (1-t)b) dt \right)^{1-\frac{1}{q}} \left[\int_0^1 (ta + (1-t)b) \left(\frac{1-t}{2} |f'(a)|^q + \frac{1+t}{2} |f'(b)|^q \right) dt \right]^{\frac{1}{q}} \\ & \leq \frac{1}{4 \times 12^{\frac{1}{q}}} \left(\frac{a+b}{2} \right)^{1-\frac{1}{q}} \left\{ [|f'(b)|^q (2a+b) + |f'(a)|^q (4a+5b)]^{\frac{1}{q}} \right. \\ & \quad \left. + [|f'(a)|^q (a+2b) + |f'(b)|^q (5a+4b)]^{\frac{1}{q}} \right\} \end{aligned}$$

The proof is completed. \square

Proposition 7. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\begin{aligned} & \left| \frac{1}{L(a, b)} - \frac{1}{H(a, b)} - \frac{1}{2A(a, b)} \right| \\ & \leq \frac{A^{1-\frac{1}{q}}(a, b)}{4 \times 12^{\frac{1}{q}}} \left\{ [b^{-2q} (2a+b) + a^{-2q} (4a+5b)]^{\frac{1}{q}} \right. \\ & \quad \left. + [a^{-2q} (a+2b) + b^{-2q} (5a+4b)]^{\frac{1}{q}} \right\} \end{aligned}$$

Proof. The proof follows from (2.3) applied to the convex function $f(x) = 1/x$. \square

Proposition 8. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\begin{aligned} & \left| L_n^n(a, b) + \frac{(n-1)G^2(a, b)L_{n-1}^{n-1}(a, b)}{2} - \frac{1}{2}A^n(a, b) \right| \\ & \leq \frac{A^{1-\frac{1}{q}}(a, b)}{4 \times 12^{\frac{1}{q}}} \left\{ \left[(nb^{n-1})^q(2a+b) + (na^{n-1})^q(4a+5b) \right]^{\frac{1}{q}} \right. \\ & \quad \left. + \left[(na^{n-1})^q(a+2b) + (nb^{n-1})^q(5a+4b) \right]^{\frac{1}{q}} \right\} \end{aligned}$$

Proof. The proof follows from (2.3) applied to the convex function $f(x) = x^n$, $n \geq 2$. \square

Proposition 9. Let $a, b \in J^\circ$, $0 < a < b$, then

$$\begin{aligned} & \left| -\ln I(a, b) + \frac{\ln(a^b/b^a)}{2(b-a)} + \frac{1}{2}\ln A(a, b) \right| \\ & \leq \frac{A^{1-\frac{1}{q}}(a, b)}{4 \times 12^{\frac{1}{q}}} \left\{ \left[b^{-q}(2a+b) + a^{-q}(4a+5b) \right]^{\frac{1}{q}} \right. \\ & \quad \left. + \left[a^{-q}(a+2b) + b^{-q}(5a+4b) \right]^{\frac{1}{q}} \right\} \end{aligned}$$

Proof. The proof follows from (2.3) applied to the convex function $f(x) = -\ln x$. \square

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